



Original Article

LOW STRAIN INTEGRITY TESTS IN PILES – 1-D AND 3-D NUMERICAL MODELING AND COMPARISONS WITH RESULTS OBTAINED IN THE FIELD

Rodrigo Paulo Strano Pasqual^{1*}, Alessander Christopher Morales Kormann¹¹ Universidade Federal do Paraná.*Corresponding author. E-mail: rodrigo@protecengenharia.com.br

INFO ARTICLE

History:

Received: 04 de julho de 2018

Accepted: 08 de setembro de 2019

Key words:

Numerical models

Finite element

Dynamics

Attenuation

Soil

Reinforcement

RESUMO

The risks associated with integrity problems in piles led to the inclusion of this matter in international standards. One of the tests available to evaluate piles integrity is known as low strain integrity test. Low costs, simple and fast execution and no waste generation are some of the main advantages of this test. However, between others, soil and reinforcement of the pile may affect the results and cause some difficulty to interpret them. In this paper two numerical models based on Finite Elements Method are presented: 1-D and 3-D. The results obtained in the numerical simulations of the tests are compared with those obtained in the field with focus in two main aspects: the signals attenuation effects and the reflections in the transition between the reinforced part of the pile and the part in simple concrete.

1. Introduction

It is important to attest the piles integrity in the field to avoid that a potential damage in a pile generates problems in part or the whole structure. One of the tests available with this objective is the low strain integrity test. In this test, the velocity variations detected in the top of the pile are recorded during a specific amount of time after striking it with a hand-held hammer. According to the one-dimensional stress-waves propagation theory, the signals characteristics are directly influenced by variations in the pile impedance. As impedance depends on the pile transversal section area and Young modulus and density of the concrete, it can be inferred that impedance reductions indicate potential damages in the piles.

Despite the advantages of the test, such as speed of execution, relatively low cost and low generation of waste, there are several variables that can affect the results and must be considered such as: maximum pile length to identify a clear toe reflection, multiple reflections caused by variations in the pile or in the stratigraphy and soil characteristics and the influence of the reinforcement in the results.

To evaluate the results obtained in the field and the influence of these variables in a more accurate way it is possible to simulate the test numerically or analytically and compare the results. Ambrosini & Ezeberry (2005) and Chai

et al. (2011) published results obtained in numerical simulations modeling the pile as a bar and the soil as a system made of dampers and springs connected in parallel and compared them with results obtained in the field under some controlled conditions. Some other papers were published in the last decade simulating the low strain integrity test in 3-D numerical models with distinct objectives and focus from those presented in this paper such as Luo et al. (2010), Ding et al. (2011), Schauer & Langer (2012), Cosic et al. (2014), Wang et al. (2014), Hetland (2015) and Hou et al. (2016).

In this paper two numerical models based on Finite Elements Method are presented: 1-D and 3-D. The results obtained in the numerical simulations are compared with those obtained in 108 piles tested in the field with focus in two main aspects: the signals attenuation effects and the reflections in the transition between the reinforced part of the pile and the part in simple concrete.

2. Finite elements numerical simulations – Theoretical Background

In this item some of the main theoretical references that form the base to simulate the test numerically are presented.

2.1 Basic principle and integration methods

The basic principle of solutions using the finite element method (FEM) is to subdivide the domain of a problem in smaller parts (finite elements) each one represented by a set of equations. The response of each element is characterized by its degrees of freedom and defined by algebraic equations (Hetland, 2015). Several softwares available on the market have been developed to numerically simulate the behavior of materials using FEM, among them: ABAQUS®, ANSYS®, SAP2000® and PLAXIS®. In these programs, sets of algorithms and methods are available for the direct integration of the structural dynamic equations in the time domain including Newmark (1959) and Hilber et al. (1977) methods or for solving the equations by successive approximations in non-linear problems such as Newton-Raphson's iterative method.

The Newmark (1959) method is based on solving the Equations 1 to 3 in each predefined time interval.

$$[M]\{a\} + [C]\{V\} + [K]\{U\} = [F(t)] \quad (1)$$

Sendo:

[M] = mass matrix;
 {a} = acceleration vector;
 [C] = damping matrix;
 {V} = velocity vector;
 [K] = stiffness matrix;
 {U} = displacement vector;
 F(t) = external loads vector.

$$v_{i+1} = v_i + [(1 - \gamma)\Delta t]a_i + (\gamma\Delta t)a_{i+1} \quad (2)$$

$$u_{i+1} = u_i + (\Delta t)v_i + [(0,5 - \beta)(\Delta t)^2]a_i + [\beta(\Delta t)^2]a_{i+1} \quad (3)$$

Sendo:

γ e β = parameters of the method;
 v_{i+1} = velocity in the moment of time "i+1",
 v_i = velocity in the moment of time "i";
 Δt = time interval between "i" e "i+1",
 a_{i+1} = acceleration in "i+1",
 a_i = acceleration in "i",
 u_{i+1} = displacement in "i+1",
 u_i = displacement in "i".

The Equation 1 is the representation of the structural dynamics general equation in its matrix form according to Newton's Second Law. The parameters " γ " and " β " define the variation of the acceleration in each time period and determine the characteristics of stability and precision of the method. The parameter " γ " is usually considered as 0,5 (no numerical dissipation) and $1/6 \leq \beta \leq 1/4$ (algorithm unconditionally stable). The Equations 1 to 3, solved in each time interval, provide the displacements, velocities and accelerations in the moment "i+1". It is evident that an iterative process is required as " a_{i+1} " (unknown value) appears in the right side on both Equations 2 and 3 (Chopra, 1995).

In the paper in which they published the method for time integration known as HHT, Hilber, Hughes and Taylor (1997) described the advantages of their method in comparison with others such as Newmark's and Houbolt and Wilson's. The process to determine accelerations, velocities and displacements in the top of the pile is based on the

solution of Newmark's Equations 2 and 3 and the solution of the Equation 4 (modified from Equation 1) (Cosic et al., 2014).

$$Ma_{i+1} + (1 + \alpha)Cv_{i+1} - \alpha Cv_i + (1 + \alpha)Ku_{i+1} - \alpha Ku_i = F_{i+1} \quad (4)$$

The parameter " α ", the only one not previously described in this paper, can be defined as a value between "0" to "-1/3" and was included by Hilber, et al. (1997) to control the stability and numerical dissipation of the algorithm. If " α " is considered to be equal to "0", the Equation 4 is the same as Equation 1 and the HHT method becomes the same as the Newmark method. In the main softwares previously listed in this paper, the parameter " α " is defined by its user.

2.2. 1-D FEM low strain integrity test simulation

Finite elements solutions can be obtained considering a one-dimensional bar in a way very similar to the one originally proposed by Smith (1960) for pile driving. The pile is considered as a bar divided in smaller elements and the soil replaced by a system of springs and dampers (Figure 1).

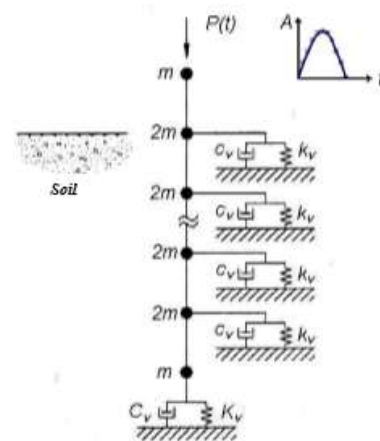


Figure 1. 1-D model for test simulation. Reference: Ambrosini & Ezeberry (2005)

The soil is simulated as a system of springs and dampers both in the pile shaft and toe with properties calculated using Equations 5 to 8.

$$kv = 2,75 \times Gs \quad (5)$$

$$cv = 2 \times \pi \times \rho s \times cs \times rp \quad (6)$$

$$Cv = \frac{0,85 \times Kv \times rp}{cs} \quad (7)$$

$$Kv = \frac{4 \times Gs \times rp}{(1 - vs)} \quad (8)$$

where "kv" is the elastic constant of the spring that represents the soil in pile shaft; "Gs" is the shear modulus of the soil; "cv" is the damping coefficient in the pile shaft; " ρs " is the density of the soil; "cs" is the shear waves velocity in the soil and "rp" is the pile's transversal section radius; "Cv" is the damping coefficient in the pile toe; "Kv" is the elastic constant of the spring that represents the soil in pile toe and "vs" is the coefficient of Poisson of the soil.

The Equations 5 to 8 are based on studies published by Lysmer and Richart (1966), Novak (1974) and Simons and Randolph (1985), and are derived assuming a rigid disk vibrating in an elastic plan, not strictly the same situation induced in the test. Yu and Liao (2006) claim that the errors introduced by this simplification are small.

The load applied in the model in the top of the pile (simulation of the impact of the hammer) is transient and generally considered varying in time as a sine function (considering only half of the wave) calculated using Equation 9.

$$F(t) = F_{max} \times \sin \pi \frac{t}{tc} \quad (9)$$

where "F(t)" is the load in the moment *f* time "t", "F_{max}" is the maximum load applied during the time in which the hammer is in contact with the pile top and "tc" is the time in which the hammer is in contact with the pile top.

The response of each element is characterized by its degrees of freedom and defined by algebraic equations (Hetland, 2015) solved using one of the direct integration methods in the time domain cited in 2.1. For each time interval displacements, velocities and accelerations are obtained in the points that define the finite elements. The velocity variation during a specific amount of time in the point defined in the top of the model can, then, be obtained numerically and compared to the reflectograms obtained in tests made in the field.

2.3. 3-D FEM low strain integrity test simulation

There are three main wave types that propagate through the pile-soil medium when striking the pile top with the hammer (compression, shear and Rayleigh waves) (Finno et al., 1997). When the problem is analyzed unidimensionally only the effects of the main compression waves are taken into account since the load is considered applied in the upper point of the pile model. Therefore, it is not possible to detect differences of velocity introduced by the strike in different points in the top of the pile, the effects arising from the relation between the hammer contact area and the cross-sectional area of the pile, nor the effects resulting from the radiation of the waves near the top of the pile.

In 3-D simulations these effects can be analyzed and the soil and the pile can be modeled in finite elements as shown in Figure 2.

Conditions of discretization in finite elements, loading, boundary restrictions, constitutive model of the soil and the conditions of contact in the soil-pile interface must be defined and are described in item 2.4. After that, displacements, velocities and accelerations in each time interval can then be obtained considering the solution of the general equation of structural dynamics and one of the integration methods previously cited.

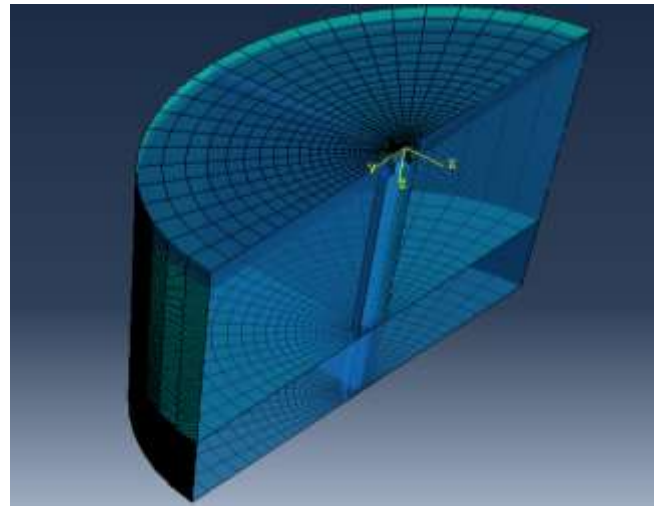


Figure 2. 3-D model for test simulation

2.4. Numerical models details

To simulate adequately the low strain integrity test it is essential that in the numerical models all elements are properly defined and sized, that the analyses occur in calculated time intervals and that the elements properties, contact and boundary conditions are defined in a way that is similar to that in the field.

2.4.1 Discretization

According to Hetland (2015) in dynamic analysis to divide the model in very large elements can result in imprecision of the results with filtration of the higher frequencies and a division in very small elements increases the computational processing time significantly. According to Kramer (1996), the maximum dimension of each element should be limited to a value between 1/8 and 1/5 of the smallest wavelength considered in the analysis. Ambrosini and Ezeberry (2005) and Cosic et al. (2014) considered a minimum division of 10 elements per wavelength.

2.4.2 Boundary conditions

When the model adopted is three-dimensional, the representation of the soil is limited in relation to the whole real medium. It is representing only a part of a mass that has continuity in practice. In the specific case of dynamic problems, adopting fixed lateral boundary conditions can result in wave reflection at the boundaries of the model that can return to the system and lead to inaccurate problem results. This concept was described by Schauer & Langer (2012), who did numerical simulations using the finite element method in conjunction with the SBFEM method that introduces "infinite" elements at the edges of the model, avoiding wave reflections at its limits that can generate interference in the results.

Peiris (2014) defined the "infinite" elements as having a behavior similar to that of "Kelvin" elements with a damper associated in parallel with a spring attached to the most distant fixed points. Hetland (2015) defined viscous elements associated with springs in the limits of his three-dimensional model elaborated in PLAXIS® 3D software with the same objective.

Another technique that can be used is to define a model with a size large enough to avoid reflections even if fixed conditions of support are attributed at the limits of the model. Wang et al. (2014) defined a model with 10m

diameter and 24 m in length for a 1 m diameter and 16 m in length pile.

2.4.3 Time stepping procedure

A very large time interval can result in an imprecise solution and a very small time interval substantially increases the calculation time as the solutions of the wave equation using the integration methods mentioned in item 2.1 are found every Δt (Hetland, 2015). As a reference, the time increment should not exceed the relation between each discretized element and the wave velocity (Fischer et al., 2010).

2.4.4 Constitutive model of the soil

The strains generated in low strain integrity tests are very small, less than 10^{-5} (Rausche et al., 1994) and caused by a transient impact. For this reason, Liao & Roesset (1997), Chow et al. (2003), Luo et al. (2010), Fekadu (2010), Lu et al. (2013), Wang et al. (2014), Yan (2015), Hou et al. (2016) defined the soil in their numerical simulations with linear elastic behavior. This consideration leads to the exclusion of damping in models. Hetland (2015) suggests predicting this factor by assigning a damping rate in the model ("Rayleigh damping") which is a linear combination of body mass and rigidity.

2.4.5 Assigning pile-soil contact conditions

Replacing the soil by springs and dampers is possible in three-dimensional models but in softwares such as ABAQUS® and PLAXIS®, the soil is usually modeled and the assignment of contact conditions at the pile-soil interface becomes necessary. The interaction between two surfaces in contact consists of two components: a normal and a tangential one. In the low strain integrity test, as the displacements are very low, it can be assumed that there is no relative slip between the pile and the soil, and therefore the assignment in the software of a rough tangential interaction (without sliding) can be assumed as well as a normal "hard" interaction (the restriction to movement is attributed when there is a contact between the surfaces and eliminated when the contact pressure becomes negative or zero) (Fekadu, 2010). Schauer & Langer (2012) considered the same, that the relative sliding between the pile and the soil can be neglected.

2.5. Data manipulation

Interpreting the results obtained in the test and in numerical simulations may be challenging but some transformations such as amplification and filtering can facilitate this task (Rausche et al., 1992). The waves generated by the impact of the hammer take a certain time to travel through the pile, to be reflected in the toe and return to the top. During this course they suffer damping with consequent attenuation of the signal received in the accelerometer. Reflections generated by pile integrity problems are also reduced due to these effects. Consequently, defects at great depths are more difficult to detect. One way to minimize this problem is to amplify the signal exponentially to the toe of the pile. In general, amplification occurs from the point where the most intense friction effects begin (Rausche et al., 1992). Care should be taken in the interpretation of amplified signals because external noises

are also highlighted using this procedure (Hertlein; Davis, 2006).

Another useful technique is filtering. Many electronic equipments have an inherent frequency response limit. The most common inherent filtering partially or totally removes the frequency components above a given value. Additional user-controlled filtering may be required when high-frequency elements make it difficult to view the low-frequency signal, which may be more important specially in the PIT test (Rausche et al., 1992).

3. FEM numerical simulations and comparisons with field results

The 1-D and 3-D models developed by the authors of this paper considering the theoretical reference indicated in the item 2 were then used to perform numerical simulations and compare them with PIT field results. For the comparisons, 5 (five) groups of piles with different characteristics, described in item 3.1, were selected in the authors database.

3.1 Characteristics of piles tested in the field

In Table 1 the main characteristics of each group of piles are summarized (diameter, length, reinforcements). In addition to the design characteristics of the piles summarized in Table 1, other data are required for input in the numerical models and subsequently compare the results obtained, such as the characteristics and elastic properties of the soil. These values were initially estimated based on the results of SPT near the piles. The modulus of elasticity and the Poisson coefficient of the soil are strongly influenced by several factors such as remolding of the soil in the tests, soil anisotropy, soil stress history, natural cementation, mass heterogeneities, stress levels and the loading speed. As the PIT is characterized by being a low strain and dynamic test, the correlation with large strain field tests such as SPT and others that mainly affect the initial layers (such as plate-bearing tests) is not accurate. Because of that we estimated initially these soil parameters based on correlations obtained from the SPT tests and subsequently compared the numerical results, within the established range of values, with the signal damping identified in the results obtained in the field. The properties for each soil layer were assigned as: Poisson Coefficient= 0.40 [average between 0.30 and 0.50 (Bowles, 1977)] and soil modulus of elasticity for low strains ($9 \times \text{NSPT}_{60}$) [average from the range of variation found by Stroud (1989) for non-drained conditions and low strains].

Lastly, the other characteristics needed are: dynamic modulus of the concrete (obtained by retroanalysis from the average wave velocities observed in the tests in each group of piles); density of the concrete of the piles (assumed to be equal to 2300 kg/m^3); Poisson's coefficient of the concrete (equal to 0.20); hammer contact time with the top of the pile ("tc") assumed to be equal to the average initial wavelength obtained by retroanalysis in the reflectograms of the tests performed on the piles in the field and the maximum load applied at the top of the pile also obtained by the average observed in the PIT reports. The procedure to consider an average of the reflectograms obtained in several blows of the hammer in the same pile is common to highlight the most important characteristics related to the integrity of the piles. A derivation of this technique was used here to highlight the average characteristic of a group of piles.

Table 1. Summary of the piles characteristics

Group	Piles			Reinforcement	
	Number of piles	Diameter (in m)	Length (in m)	Number of bars and diameter (in mm)	Length of the reinforcement barsembedded in concrete (in m)
1	40	0.50	13.00	6 de 16	8.00
2	32	0.30	15.00	4 de 16	4.60
3	12	0.50	10.90	5 de 16	8.00
4	12	0.70	20.50	5 de 32	8.80
5	12	0.80	21.80	12 de 25	10.50

3.2. Comparisons between numerical simulations and reflectograms obtained in the field

Assigned in the numerical models the properties and characteristics of the piles, soil and impact indicated in the item 3.1, the reflectograms (velocity x time) were obtained and amplified by an exponential function similar to that adopted in the treatment of the results obtained in the field. In the numerical models validation processes it was identified similarity between the results obtained one-dimensionally and three-dimensionally when the simulated position of the accelerometer was chosen near the edge of the pile (minimum 7 cm and maximum 15 cm). In the 3-D simulations the computational effort and time processing are significantly higher. To illustrate that, in one case, the processing time for the one-dimensional model was about 90 times lower than that of the 3-D model. In this research all simulations led to the conclusion that a pile can be considered, by approximation, a one-dimensional elastic bar in a low strain integrity test simulation if it satisfies two conditions: the wavelength is greater than the diameter and smaller than the length of the pile, the same obtained by Luo et al. (2010).

As each of the groups of piles 1 to 5 is composed of several piles, the reflectograms obtained in the field were superimposed on each other to highlight the average signal characteristic. In the Figure 3 the higher and lower limits obtained in the field tests are indicated.

The force applied in the hammer blow and the time in which the hammer was in contact with the top of the pile were calibrated according to average conditions observed in the tests made in each specific group of piles. In Figure 4 the comparison between the results obtained in the numerical simulations and the results obtained in the field in specific piles of each groups is shown to emphasize the similarity between them.

From the comparative analyzes carried out, it is possible to draw some conclusions, reported in item 4, regarding the prediction of the damping generated by the soil, the influence of the reinforcement embedded in piles and also the prediction of the propagation velocity of the tension waves.

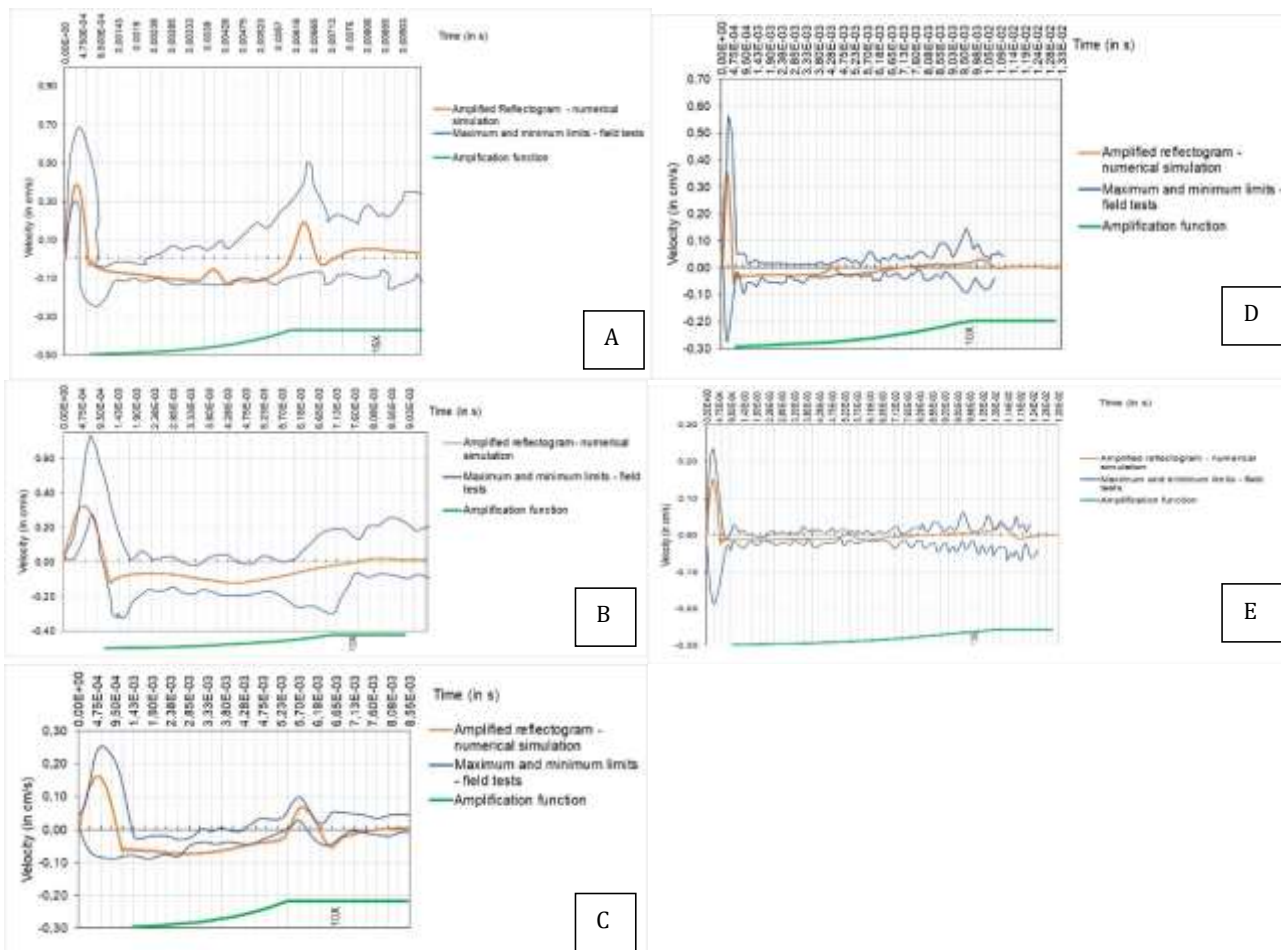


Figure 3. Comparison between numerical simulation and field results – piles from Group 1 to 5 (A to E)

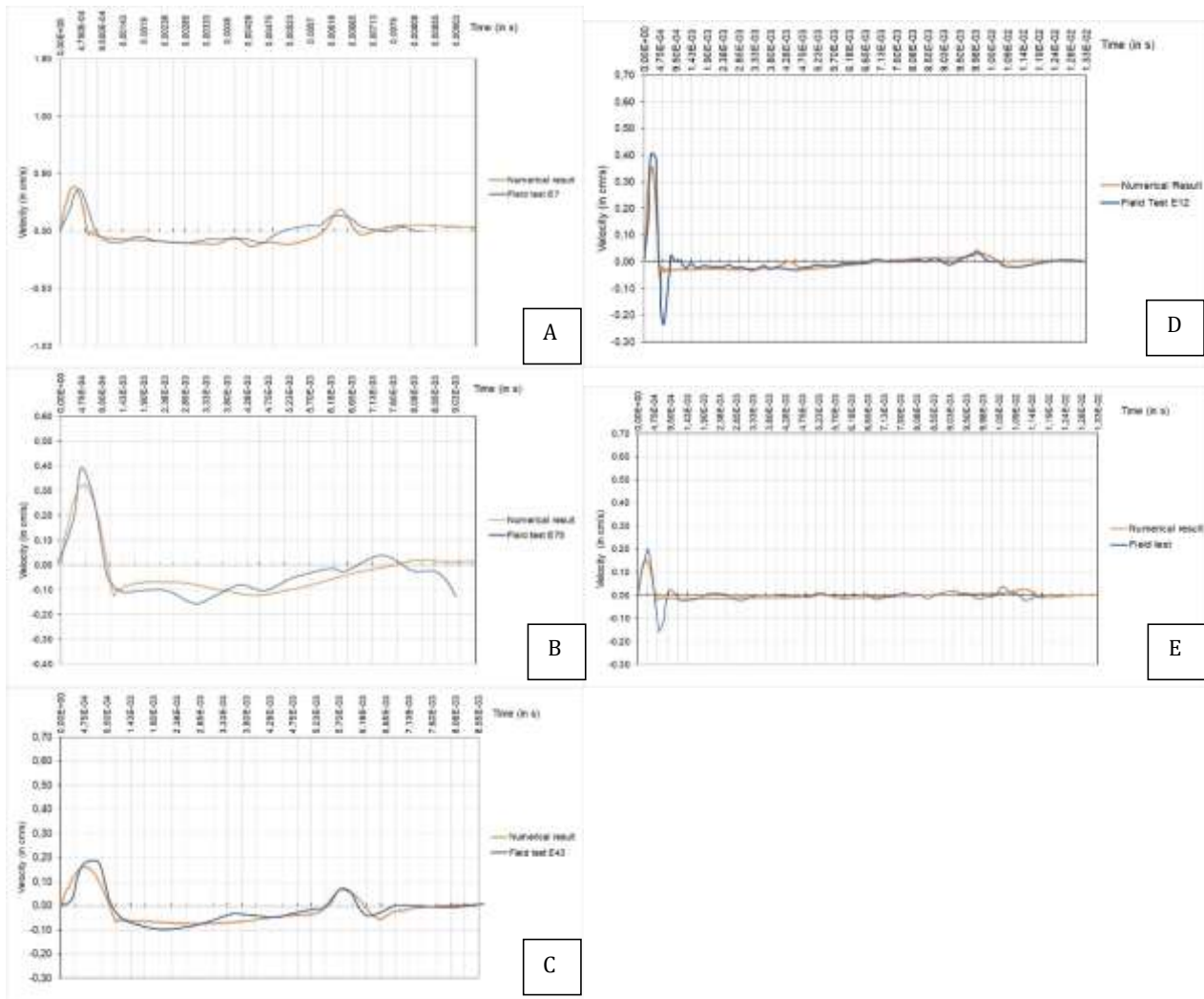


Figure 4. Comparison between numerical simulation and field results – pile E7 from Group 1 (A), pile E75 from Group 2 (B), pile E43 from Group 3 (C), pile E12 from Group 4 (D), pile E9 from Group 5 (E)

4. Conclusions

The attenuation of the signals obtained in the simulations, based on damping coefficients and soil elastic constants estimated from the theory of a hard disk vibrating in an elastic plane and calculated considering an estimative of the soil modulus of elasticity based on correlations with SPT results, was satisfactory. The results indicate that the damping verified in the field tests approach that observed in the numerical simulations.

As the prediction of the damping in the numerical models was similar to that observed in the field, it is possible to conclude that, with SPT reports and adopting correlations with the soil modulus of elasticity at low strains, it is possible to forecast for a specific condition, what would be approximately the pile limit L/D ratio (length divided by the diameter) so that the low strain integrity test is applicable, prior to its realization. In the simulations performed this limit ratio was found to be approximately equal to 30. It is evident in the Figure 3B (group 2 of piles) that the toe reflection was not clearly identified. The relation L/D of that piles is 50. In Groups 4 and 5 (L/D equal to 29 and 27, respectively) the toe reflection was clear although very small. If the need for amplification of the toe signal obtained in the numerical simulation is very high, the field test will probably not offer satisfactory results. The authors of the present research recommend a limit of about 25 times for the amplification. The reason to adopt this limit is that in simulations carried out, when the factor adopted was superior to this one, some

distortions in the reflectogram were observed emphasizing intermediate reflections and effects of the soil making it difficult to interpret the results. It is important to emphasize that in cases where there are many intermediate reflections and loss of energy of the impact along the shaft by factors not included in the numerical simulation the actual damping will be larger than expected numerically.

In the piles tested in the field in which the signal generated by the blow of the hammer indicates a lower average contact time at the top of the pile (piles of groups 1, 4 and 5) reflectograms shows more high frequency interference and greater difficulty of identification of the toe reflection in comparison to the piles with longer contact time (groups 2 and 3), corroborating with the conclusions of Klingmüller and Kirsch (2004) and Plassmann (2001). The choice of material and weight of the hammer, therefore, will influence the results (frequency spectrum and external interferences) and is primordial to obtain good results that allow the analysis of pile integrity.

In most of the piles (Groups 1, 3, 4 and 5) it was not clearly identified in the field a reflection in the end of the reinforcement cage. That reflection was expected due to the variation of the modulus of elasticity and density between the reinforced and non-reinforced parts of the pile. High frequency interferences, higher damping of the signals at higher depths and the relatively low reinforcement rate of the piles (up to 1.17%) contributed to make it difficult to identify this reflection. In some piles with smaller diameter (0.30 m), with clear identification of the toe reflection, with no signal

reflections identified before the tip of the reinforcement bars embedded in concrete and with a transition between reinforced and non-reinforced parts of the pile at a lower depth (up to 4.60 m) it was possible to identify velocity peaks near the reinforcement tip. That indicates an impedance variation that can be attributed partially to the effect of the reinforcement embedded in the concrete of the pile and the possible greater densification of the concrete in the region of the reinforcement tip.

Comparing the compression wave velocities obtained in the tests in the field (obtained by dividing double the length of the pile by the elapsed time between the hammer strike and the reception of the pile toe reflection) with those obtained by correlations with the possible dynamic modulus of elasticity of the concrete (calculated from f_{ck} prediction) differences were found between -17.10% and 56.01%. In a numerical simulation of a PIT test it is advisable to calibrate this concrete property by retro analysis or by making a previous test in the field using a pair of accelerometers placed at a known distance in the pile shaft.

5. Acknowledgements

The authors are pleased to the Civil Engineering Post-Graduation Program (PPGECC) from de Federal University of Paraná (UFPR) for all the support to making this research possible and to the company Protec Engenharia de Projetos to provide all the field tests reports used.

6. References

- Ambrosini, D., Ezeberry, J. (2005). Long piles integrity through Impact Echo technique. In Proceedings of the 8th Congreso Argentino de Mecánica Computacional, Buenos Aires, Argentina, 2005 (pp. 651-669).
- Bowles, J. E. (1977). *Foundation analysis and design*. New York: McGraw Hill, Ltda.
- Chai, H.Y., Wei, C.F., Phoon, K.K., Yang, Y.M. (2011). Some observations on the performance of the signal matching technique in assessment of pile integrity. *Journal of Nondestructive Evaluation*, 30(4), 246-258.
- Chopra, A. K. (1995). *Dynamics of structures: Theory and Applications to Earthquake Engineering*. New Jersey: Prentice Hall.
- Chow, Y. K., Phoon, K.K., Chow, W.F., Wong, K.Y. (2003). Low Strain Integrity Testing of Piles: Three-Dimensional Effects. *Journal of Geotechnical and Geoenvironmental Engineering*, 129(11), 1057-1062.
- Cosic, M., Folic, B., Folic, R. (2014). Numerical simulation of the pile integrity test on defected piles. *Acta Geotechnica Slovenica*, 11 (2), 4-19.
- Ding, X., Liu, H., Liu, J., Chen Y. (2011). Wave propagation in a pipe pile for low-strain integrity testing. *Journal of Engineering Mechanics*, 137 (9), 598-609.
- Fekadu, P. (2010). *Simulating the dynamic response of a soil-pile system using ABAQUS*. (Dissertação). University of Technology, Göteborg, Suécia.
- Finno, R. J., Gassman, S. L., Osborn, P. W. (1997). Non-destructive evaluation of a deep foundation test section at the Northwestern University national geotechnical experimentation site. In A Report Submitted to the Federal Highway Administration Office, Northwestern University, Evanston, Illinois, 1997.
- Fischer, J., Missal, C., Breustedt, M., Stahlman, J. (2010). Numerical simulation of low-strain integrity tests on model piles. In Benz & Nordal (Eds), *Numerical Methods in Geotechnical engineering* (pp. 655-660). London: Taylor and Francis Group.
- Hertlein, B., Davis, A. (2006). *Nondestructive Testing*. Chichester, West Sussex, Inglaterra: John Wiley & Sons Ltd.
- Hetland, J. (2015). *Numerical Modelling of a Pile Model Test with Focus on Small-strain Stiffness*. (Dissertação). Norwegian University of Science and Technology, Noruega.
- Hilber, H. M., Hughes, T. JR., Taylor, R. L. (1977). Improved numerical dissipation for time integration algorithms in structural dynamics. *Earthquake Engineering & Structural Dynamics*, 5 (3), 283-292.
- Hou, S.W., Hu, S.J., Guo, S.P., Zeng, Y.Q. (2016). The Research of Multi-defective Piles for Low Strain Testing and Numerical Simulation. In *The 2016 Structures Congress*, Jeju Island, Coréia.
- Klingmüller, O., Kirsch, F. A. (2004). Quality and Safety Issue for Cast-in-Place Piles-25 Years of Experience with Low-Strain Integrity Testing in Germany: From Scientific Peculiarity to Day-to-Day Practice. In *Current Practices and Future Trends in Deep Foundations*, (pp. 202-221).
- Kramer, S. L. (1996). *Geotechnical earthquake engineering*. India: Pearson Education.
- Liao, S. T., Roesset, J. M. (1997). Dynamic response of intact piles to impulse loads. *International Journal for Numerical and Analytical Methods in Geomechanics*, 21 (4), 255-275.
- Lu, Z. T., Wang, Z. L., Liu, D. J. (2013). Study on low-strain integrity testing of pipe-pile using the elastodynamic finite integration technique. *International Journal for Numerical and Analytical Methods in Geomechanics*, 37 (5), 536-550.
- Luo, W., Chen, F., Hu J. (2010). Improvement of Low Strain Pile Integrity Test. In *Proceedings of the ASEE (American Society of Engineering Education) Conference*, Beijing, China, 2010 (pp. 583-589).
- Lysmer, J. F. E. R., Richart, F. E. (1966). Dynamic Response of Footings to Vertical Loading. *Journal of the Soil Mechanics and Foundations Division*, 92(1), 65-91.
- Newmark, N. M. (1959). A method of computation for structural dynamics. *Journal of Engineering Mechanics*, ASCE, 85 (3), 67-94.
- Ni, S.H., Lehmann, L., Charng, J. J., LO, K.-F. (2006). Low-strain integrity testing of drilled piles with high slenderness ratio. *Computers and Geotechnics*, 33 (6), 283-293.
- Novak, M. (1974). Dynamic stiffness and damping of piles. *Canadian Geotechnical Journal*, 11(1), 574-598.
- Ordóñez, J. A. R., Botero, V. H. R. (2010). Análisis espectral de propagación de ondas para análisis de pruebas de integridad de pilotes. *Ingeniería y Universidad*, 10 (2), 1-10.
- Peiris, T. P. (2014). *Soil-Pile Interaction of Pile embedded in Deep Layered Marine Sediment under Seismic Excitation*. (Tese de doutoramento). Queensland University of Technology, Brisbane, Austrália.
- Plassmann, B. (2001). *Zur Optimierung der Messtechnik und der Auswertemethodik bei Pfahlintegritätsprüfungen*. (Tese de doutoramento). Universidade Técnica Carolo-Wilhelminazu Braunschweig, Nürnberg, Alemanha.
- Rausche, F., Likins, G., Hussein, M. (1994). Formalized procedure for quality assessment of cast-in-place shafts using sonic pulse echo methods. *Transportation research record*, 1 (1), p. 30-30.
- Rausche, F., Likins, G., Kung, S. R. (1992). Pile integrity testing and analysis. In *Proceedings of the 4th International Conference on the Application of Stress-Wave Theory to Piles*, Holanda, 1992 (pp. 613-617).
- Schauer, M., Langer, S. (2012). Numerical simulations of pile integrity tests using a coupled FEM SBFEM approach. *PAMM*, 12 (1), 547-548.

- Simons, H.A., Randolph, M.F. (1985). A New Approach to One-Dimensional Pile Driving Analysis. In Proceedings of the 5th International Conference on Numerical Methods in Geomechanics, Nagoya, Japão, 1985 (pp. 1457-1464).
- Smith, E.A.L. (1960). Pile-Driving Analysis by the wave equation. ASCE Paper, 127(3306), part I.
- Stroud, M. A. (1989). The standard penetration test - its application and interpretation. In Proceedings of the Conference on Penetration Testing in the UK, Londres, Inglaterra, 1989.
- Wang, Z., Wu, Y., Xiao, Z. (2014). Numerical Assessment of Factors Affecting Waveform Based on Low Strain Testing of Piles. Open Civil Engineering Journal, 8 (1), 64-70.
- Yan, N. (2015). Numerical Modelling and Condition Assessment of Timber Utility Poles using Stress Wave Techniques. (Tese de doutoramento). University of Sidney, Austrália
- Yu, C.P., Liao, S.T. (2006). Theoretical basis and numerical simulation of impedance log test for evaluating the integrity of columns and piles. Canadian geotechnical journal, 43 (12), 1238-1248.