

# The technique to solve equations of the first degree. A case study

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**ABSTRACT.** In Mexico we can find upper secondary teachers who, sometimes, are forced to teach mathematics even though their profile is different. This document is aimed to identify the technique – mathematical methods– that a teacher from the Mexican Telebachillerato subsystem (high school) uses to help students to solve the type of tasks related to first-degree equations. This proposal is part of a research project where thirteen teachers participated, and who allowed to be observed twice teaching mathematical contents according to their study program. The purpose of observation works was to make them a discussion object with the participants. This is a study case research. The results show that the teacher creates a technique focused on obtaining determinants to solve types of task about system of equations, but the theory which proves its functionality is reduced to a more didactic than mathematical discourse. In the classroom, techniques to solve problems are taught, but these remain unprotected from mathematical theories (mathematical argument) that can validate their functionality, the student is the one who has to infer or build his own theory from the teacher’s technique.

**Keywords:** Teaching practice; mathematics; algebra; high school

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## BACKGROUND

The research on teaching practice shows the significance of understanding the relationship between the disciplinary knowledge and its didactics, due to its impact on the student’s learning (Wilkins, 2008). According to Alder, Ball, Krainer, Lin and Novotna (2005) and the National Council of Teachers of Mathematics (NCTM, 2014), it is crucial for the teacher to develop acceptable didactic skills and the necessary disciplinary knowledge. In Mexico, the teaching staff for mathematics in upper secondary education (high school) is conformed, mainly, by professionals from areas related to mathematics without a teaching training (Montiel, & Castañeda, 2009). Occasionally, the teaching training of these educational actors is a consequence of beliefs on how to teach and learn (Guzmán, 2001). This situation gets worse in institutions located in an unfavorable context, for

example, the subsystem Telebachilleratos (high school) where students’ population belongs to a low socioeconomic level, the teachers use to have little or no experience in teaching curricular contents (Instituto Nacional de Evaluación para la Educación [INEE], 2016, 2017). Related to this, Telebachillerato teachers have the responsibility of teaching mathematics even though their profile differs with the subject; in addition, he has the responsibility of design strategies that make knowledge relevant. In this way, he favors the student to take control of his learning process. It is necessary to promote “... questioning so that the student knows and reflects on the learning strategies that he himself uses to improve” (Secretaría de Educación Pública [SEP], 2017, p. 88).

This education subsystem arose due to the limited educational attention in communities with a high level of margination (Aguilar, Gamboa, & Farias, 2015). For young people who study in this subsystem, the school and the classroom represent the ideal place to offer them opportunities that allow them to improve their condition; on the other hand, for the teacher, it is a challenge to teach in Telebachillerato, since they have to adapt their activities to the students’ conditions and those of the school (INEE, 2015, 2016). The teacher must design learning environments

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where students can build mathematical knowledge through challenging situations, as well as "... [he] facilitate the educational process by designing meaningful activities that promote the development of competences (knowledge, skills and attitudes), [in addition, he must] enhance the role of students as autonomous managers of their own learning" (Subsecretaría de Educación Media Superior [SEMS], 2017, pp. 5-6). However, the lack of educational achievement and the last national and international evaluations low results show that it is necessary to approach to the teaching practice in this educational subsystem to understand what and how mathematics is taught, under what criteria learning is promoted in conditions of poverty.

In Mexico, research on the upper secondary teacher is related, mainly, to teacher training and development programs (Martínez-Rizo, 2013). Studies related to the mathematics teacher's practice in this education level are focused on the content knowledge to teach or on the problems solving in pencil-and-paper and technological environments (Sosa, Flores-Medrano, & Carrillo, 2014). Nowadays, studies related to the teaching practice in this level are needed, it is clear that the upper secondary teacher is a key element to set a connection between Elementary school and High school, but teachers, who work in schools located in suburban, rural, or poor zones, have bigger challenges due to the actual conditions (INEE, 2016).

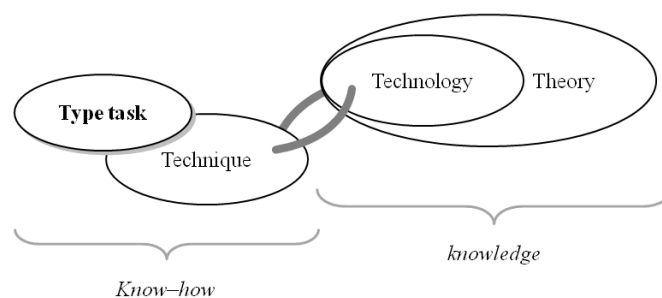
Regarding the idea above, this document is aimed to identify the construction and validation of the mathematical praxeology technique that the Telebachillerato teacher proposes in the classroom; namely, to give an account of the mathematical technique that is taught and how it is put into practice. The interest in studying the Telebachillerato teacher arises because it is a field little addressed and due to its importance in the mathematics' teaching-learning process, the purpose is to comprehend what the teacher does and says inside the classroom to give mathematics a meaning formed in classes of an everyday environment.

## REFERENCE FRAMEWORK

The reference framework for this study is that of *mathematical* and *didactic praxeologies* of Chevallard's *Anthropological Theory of Didactics* (1999, Bosch & Gascón, 2009, Castela, 2016, 2017, Chevallard, Bosch, & Gascón, 1997). This theory is suitable for research in which a problem is analyzed in the field of mathematical education, and whose purpose is to contribute to the progress of didactics in the field of mathematical knowledge, particularly algebra. In addition, the study conforms to this anthropological approach because we take into account the professor as part of the research team (Bosch & Gascón, 2009, Castela, 2016), and not just as an object of study.

The Anthropological Theory of Didactics emerges within the anthropology of mathematics, that is, from the production of mathematical knowledge originating in the anthropology of epistemology, which gives space to the "study of man doing mathematics" (Chevallard, 1999, p. 26). The Anthropology of the Didactic is a theory which allows having an approach to the mathematics teacher's practice and knowledge (Chevallard, 1999, Chevallard, Bosch y Gascón, 1997, Castela, 2016, 2017). This proposal establishes that mathematics is constructed by a group of participants (for example, the teacher and the students) through the practice within an institution. For Chevallard et al. (1997, p. 51), "... an important aspect of the mathematical activity lies in the construction of a model (mathematical)..., work with this model and interpret the results obtained... to answer to the problems [to solve tasks] suggested at the beginning. Much of the mathematical activity can be identified, thus, as an activity of mathematical modeling."

The mathematical activity –and the process of mathematics study– is part of the set of human activities and those of the social institutions (Chevallard, 1999). This activity can be defined in terms of *Praxeology* where *praxis* means practice and *logos* means knowledge. Both terms are related with each other: There is not praxis without logos, but neither logos without praxis" (Chevallard et al., 1997, p. 274). The praxeology is seen as the practice that emerges with the knowledge and it is composed by four elements: *Type task* (Tipo de tareas), *Technique* (Técnica), *Technology* (Tecnología) and *Theory* (Teoría) (Figure 1).



**Figure 1.** Parts of a praxeology as a mathematical activity constructed by a community (Chevallard, 1999). See also Páez, Eudave, Cañedo & Macías (2020, p. 806).

The origin of every praxeology is the task, Chevallard (1999) mentions that the task is said through a verb: calculate, integrate, simplify, determine, factoring, among others. A verb can point out one or many tasks, but one *Type of task* means a specific one; for example, to determine the values of  $x$  in the equation  $x^2+3x=5$  or to determine the condition or conditions at  $x$  to get the inequality  $2\leq 3x<35$ . The *Type of task* is an institutional construction that

emerges from the practice and it is the basis of all praxeology, since from it the other elements are given. To carry out the Type of task, it is necessary, at least, of a way of solving it, in a systematized and safe way (Chevallard et al., 1997). This way is called *Technique* and does not necessarily refer to an algorithmic type, they can be many techniques for a type of task; however, in an institution sometimes just one can be recognized, blocking some others that can be similarly effective. The Technique implies a rational discourse that justifies its applicability and validity, defined as Technology and it has three functions: (a) to justify the technique to make sure it works, (b) to explain the technique in order to clarify it, and (c) to make new techniques. The technological discourse has statements that have to be rationally justified too, for Chevallard (1999) this justification is the *Theory*.

The four elements that make all praxeology up are organized in two inseparable blocks: the practical-technical block which determines a *Know-how* (Saber-hacer), and the *theoretical-technological block* which refers to a *knowledge* (Saber) (Figure 1). The mathematical praxeology comes out as a result of a didactic process (study of mathematics) that is carried out by a subject or a study community within an institution.

## MATERIAL AND METHODS

This an explorative research through case studies (Álvarez, 2003, Schoenfeld, 2007, 2008). Because of the study's nature, thirteen teachers who teach mathematics in different Telebachillerato schools participated, these schools are located in Mexico. Most of the participants hold a postgraduate degree in education, particularly, two of them hold a master's degree in mathematics teaching; moreover, the half of the participants have, at least, eight years of teaching experience in this subsystem. Each teacher, prior authorization, was observed (video recording) when teaching mathematics to his students according to Telebachillerato's study program, with the objective of making his practice object of discussion through reflecting on it (Páez, Ramírez, Cañedo, Eudave, Carvajal & Macías, 2019, Páez et al., 2020).

Two classes per teacher at different moments were observed (Table 1). It was a non-participant observation in order to explore the classroom's reality (Adler & Adler, 1994) and therefore, "... to have the opportunity of gathering data 'straight' from situations that occur in a natural way" (Cohen, Manion & Morrison, 2007, p. 396).

**Table 1.** Teachers and content worked in the observed classes

Teacher	Mathematical Content	
	First class	Second class
Gerardo	First-degree equations.	-
Luis	Ellipse.	Functions (Concept).
Paco	First-degree equations.	Pythagoras Theorem.
Carlos	Special products.	-
Ana	Factoring.	-
Julio	First-degree equations.	-
Juan	Determinants 2x2 matrix.	Pythagoras Theorem.
Rosa	Addition and subtraction of polynomials.	Complementary angles.
Cecilia	Circle Characteristics.	-
Miguel	Special Products.	Complementary angles.
Pedro	Special Products.	Complementary angles.
Carla	-	Complementary angles.

Source: Páez et al. (2019).

In a first moment, eleven classes were video-recorded, in the second observation just seven. Some teachers who participated in the first observation decided not to do it in the second one. The class observations were carried out between November 2012 and February 2018, in order to do it, the date of video-recording was determined with the teachers, the study program was respected (content to teach and the needed time for it). Even though, video cameras were used to observe, it was intended to respect the stage and the natural context the teaching-learning process given at that moment. The observations were on the mathematics content that the teachers were teaching at that time according to their Curriculum (SEP, 2015), so some of the mathematic content is different in the two

moments of the video-recorded classes, although some teachers coincided the same math content.

The analysis of the classes was centered in identifying the four components of the constructed praxeology by the teachers in the two observed classes, and how these elements are related each other. One of the most relevant mathematics contents in the observed classes is that of first-degree equations, this mathematical content was taught by Gerardo, Paco and Julio in the observed classes (Table 1). According to the data collected, Julio had a greater participation in the class and interaction with his Telebachillerato students. For this document, a case study is presented: teacher Julio, teaching first-degree equations to first-semester Telebachillerato students, who show the

construction and validity of the technique to solve lineal equations.

## RESULTS ANALYSIS

The textbook is an essential resource for the Telebachillerato teacher, due to it provides to him the three first praxeology elements: types of task, techniques and technology for the first-degree equations topic (SEP, 2015). The textbook suggests four techniques to solve the specific types of task around this mathematical content: method of equalization, determinants, reduction and graph (Cfr. SEP, 2015). In the classes observed, the praxeology constructed by Julio, was centered in working the determination and reduction methods. For the types of tasks  $ax+by+c=0$ , Julio shows the technique that students must use, which is centered in using the coefficients of the equations (Figure 2).

$$\begin{array}{l} \downarrow \\ \begin{array}{l} ax + by = c_1 \\ ax + by = c_2 \\ a_1 \quad b_1 \quad | \quad c_1 \\ a_2 \quad b_2 \quad | \quad c_2 \end{array} \end{array} \quad \begin{array}{l} 2y - 6x = 4 \\ y - 3x = 2 \\ \downarrow \quad \downarrow \\ \begin{array}{l} 2 \quad -6 \quad | \quad 4 \\ 1 \quad -3 \quad | \quad 2 \end{array} \end{array}$$

Figure 2. Type of task: Calculate the values of unknown in the first-degree equation system.

The technique is centered in obtaining three determinants, which are defined by Julio as  $D_p$  (Main determinant),  $D_a$  (Coefficient's determinant a) and  $D_b$  (Coefficient's determinant b). To obtain each determinant, Julio suggests a cross product between the first equation's coefficient and the second equation's coefficient (Figure, 3). The relation involves additions and subtractions of the results. The construction of the technique through determinants leads to identify the relations between the cross pairs of the coefficients to obtain a third one.

$$\begin{array}{l} \downarrow \\ \begin{array}{l} \textcircled{1} \dots ax + by = c_1 \\ \textcircled{2} \dots ax + by = c_2 \\ a_1 \quad b_1 \quad | \quad c_1 \\ a_2 \quad b_2 \quad | \quad c_2 \end{array} \end{array} \quad \begin{array}{l} 2y - 6x = 4 \\ y - 3x = 2 \\ \downarrow \quad \downarrow \\ \begin{array}{l} 2 \quad -6 \quad | \quad 4 \\ 1 \quad -3 \quad | \quad 2 \end{array} \end{array}$$

$$DP = (a_1 b_2) - (b_1 a_2)$$

$$D_a = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} D_a = (c_1 b_2) - (b_1 c_2)$$

$$D_b = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} D_b = (a_1 c_2) - (c_1 a_2)$$

$$x = \frac{D_a}{DP} \quad y = \frac{D_b}{DP}$$

Figure 3. Technique: Cross relation between pairs of equations' coefficients.

In this technique, Julio emphasizes that  $D_a$  and  $D_b$  will allow to obtain the values of unknowns  $x$  and  $y$  by dividing the result of the relation with the value of  $D_p$ . It is clear that Julio's discourse, centered in the solving process, is a theory that supports the functionality of this technique. Julio explains step by step how to develop the technique without giving an explanation of it. The technique first involves  $D_p$ , to obtain later the other two determinants. In the construction of the technique developed by Julio, at the same time, it can be seen the theory which gives meaning to it.

The theory suggested by Julio is related to the lining up of the coefficients of the equations according to the laterals which refer to the columns and rows taking  $a$ ,  $b$ , and  $c$ . Julio states that it is essential the lining up of the coefficients to be able to use the technique, he says to the students: "if you pay attention, columns and rows are created. This column [the first one] refers to the letter  $x$ , this column [the second one] is the letter  $y$ , and the other column is the letter  $c$ . A second theory, given by Julio too, is the data arrangement to obtain the values of the determinants, Julio suggests that "it is important to take the equations data from left to right" and "from right to left" as shown in Figure 3. It seems that the justification sets aside the commutative law which suggests that numbers can be swapped and still get the same result; that is,  $a_1 b_2 = b_2 a_1$ .

## CONCLUSIONS

The results show that Julio is really interested in helping students to understand how to solve first-degree equations; for that, he suggests the technique of determinants according to the study program (Páez et al., 2020). The technique approaches to the institutional mathematic content; it is a procedure that involves two steps: obtaining the determinants and later generating divisions. Julio's theory is more of a didactic type, in terms of the students making a correct use of the technique: to arrange the data such a way that the technique works.

The theory's construction is far away from the mathematical discourse, such as proving and arguing the technique in mathematical way, in this sense the gaps of why it is necessary to obtain a main determinant and which is the technique's functionality are left. The students have to build or infer the mathematical theory which is implied within Julio's technique, for example, that the main determinant is necessary and it plays an important role to obtain the values  $x$  and  $y$  by dividing other determinants and this one. Perhaps, the lack of mathematical theory is due to Julio follows the study program, whose objective is to teach techniques (SEP, 2015).

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## DECLARATION OF COMPETING INTEREST

None declared.

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